The vector coupling in IR region from splittings in bottomonium

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Abstract

The splittings $1D-1P, 1F-1P, 2D-2P$ are shown to be the most convenient characteristics to determine the vector coupling in IR region. In background perturbation theory the splitting $\Delta = 1D-1P$ appears to be in agreement with $\Delta(\exp) = 261.1 \pm 2.2$ MeV only for large freezing value, $\alpha_{crit} \cong 0.60$, which corresponds to $\Lambda_{\overline{MS}}$(2-loop, $n_f = 5$) $\cong 240$ MeV ($\alpha_s(M_Z) = 0.1193(2)$. The masses $M_{cog}(1F) = 10362 \pm 3$ MeV and $M_{cog}(2D) = 10452 \pm 4$ MeV are predicted.

1. The static potential plays a special role in heavy quarkonia physics. Although $V_{st}(r)$ was introduced by the Cornell group 30 years ago [1], even now we do not fully understand some important features of static interaction in QCD, moreover uncertainties refer both to perturbative (P) and nonperturbative (NP) contributions to $V_{st}(r)$. There are three characteristic features of static potentials, widely used in QCD phenomenology:
1.additivity, when $V_{st}(r)$ is taken as a sum of confining and the gluon-exchange terms:

$$V_{st}(r) = V_{NP}(r) + V_{GE}(r); \tag{1}$$

2.linear behavior of $V_{NP}(r) = \sigma r$ over the whole region of $Q\bar{Q}$ separations r;
3.constant value of the vector coupling $\alpha_V(r)$ at large r [2, 3]. (By definition $V_{GE}(r) = -\frac{4}{3}\frac{\alpha_V(r)}{r}$). In some cases $\alpha_V(r)$ =const. is taken already at not large r ($r \gtrsim 0.2$ fm), as in lattice QCD [4], or even at any separations r as in the Cornell potential [5]. Last assumption can be justified only if the "true" vector coupling freezes at rather small r, while asymptotic freedom behavior

is supposed to be inessential in first approximation, as for high excitations in charmonium.

Although for long time "the freezing" of $\alpha_V(r)$ is widely used in QCD phenomenology, nevertheless, till now there is no consensus about the true value of freezing (or critical) constant α_{cr}. In different approaches the values of α_{cr} vary from $\alpha_{cr}(lat) \approx 0.23 \div 0.30$ in lattice QCD [4], the values 0.39-0.45 for the Cornell potential [5] up to the number $\alpha_{crit} \approx 0.60$ in background perturbation theory (BPT) [3, 6] and also in the famous paper [2]. Even larger values, $\alpha_{eff}(1GeV) \cong 0.9 \pm 0.1$, were determined from the hadronic decays of the τ-lepton [7] and in analytical perturbation theory (APT) [8], where $\alpha_{cr}(n_f = 3) \cong 1.4$.

Some achievements in our understanding of static interaction on the fundamental level mostly refer to NP term. In particular, the property of additivity has been confirmed by lattice calculations of static potentials in different group representations where the behavior $V_{st}(r, N) = C_F \tilde{V}_{st}$ (universal) ($r \lesssim 1$ fm), or the Casimir scaling property, has been observed in [9], and in Ref. [10] theoretical interpretation of the Casimir scaling has been given.

With the use of the vacuum correlators, measured on the lattice [11], it was shown that linear behavior of $V_{NP}(r)$ takes place only in the range $T_g \lesssim r \lesssim R_{SB}$ [12]. Here $T_g \approx 0.2$ fm is the gluonic correlation length [11], while $R_{SB} \approx 1.2$ fm characterizes those separations $r > R_{SB}$, where the string breaking is essential [13, 14] and linear potential is becoming more flat. Such flattening of confining potential does not affect the positions of the $b\bar{b}$ levels, which lie below $B\bar{B}$ threshold, but this effect is essential for the states of large size : $\sqrt{\langle r^2 \rangle_{nL}} \gtrsim 1.2$ fm. In particular, in light meson sector this effect provides a correlated large shift (down) of radial excitations like $\rho(3S), \rho(4S), a_J(2P)$ [14].

At present there is no theory of string breaking and also we do not know precise behavior of $V_{NP}(r)$ at small r. Meanwhile, knowledge of $V_{NP}(r)$ at small r is very important: (1) for understanding of fine structure splittings of χ_b mesons (through the Thomas precession term) [15]; (2) for explanation of very small shift of $h_c(1^1P_1)$ with respect to $M_{cog}(1^3P_J)$ in charmonium, where a cancellation of two small terms–negative P term and positive NP term takes place [16].

Here we concentrate on the gluon-exchancge term. A unique information about the vector coupling $\alpha_V(r)$ can be extracted from the analysis of the splittings between high excitations (still lying below $B\bar{B}$ threshold) in bot-

tomonium. There are several reasons for that. First, the splittings between $b\bar{b}$ levels are known from experiment with precision accuracy, $\delta M \lesssim 1$ MeV. Second, there are ten observed (plus unobserved $1F, 2D, 3D$, and may be $3P$) states which lie below $B\bar{B}$ threshold. These states have very different r.m.s. radii, which spread from 0.2 fm for $\Upsilon(1S)$ up to 0.8 fm for $2D$ and $3P$ states (see Table 1).

Table 1: The r.m.s. radii $\sqrt{\langle r^2\rangle_{nL}}$ in bottomonium from [6]

state	1S	1P	2S	2P	1D	1F	3S	2D	3P
$\sqrt{\langle r^2\rangle_{nL}}$ in fm	0.22	0.38	0.46	0.62	0.62	0.63	0.71	0.73	0.82

In our analysis of the $b\bar{b}$ spectrum we use $V_B(r) = \sigma r - \frac{4}{3}\frac{\alpha_B(r)}{r}$ where the vector coupling $\alpha_B(r)$ is defined as in BPT [3, 6],

$$\alpha_B(r) = \frac{2}{\pi}\int_0^\infty \frac{dq}{q}\sin(qr)\alpha_B(q), \tag{2}$$

while the background coupling in momentum space is given by the standard formula:

$$\alpha_B(q) = \frac{4\pi}{\beta_0 t_B}\left(1 - \frac{\beta_1}{\beta_0^2}\frac{\ln t_B}{t_B}\right) \tag{3}$$

with the modification of the logarithm:

$$t_B(q) = \ln\frac{q^2 + M_B^2}{\Lambda_V^2}, \tag{4}$$

where $M_B = 2.24(1)\sqrt{\sigma}$ is so called background mass, defined by the lowest hybrid excitation and expressed through the string tension [3]. The QCD constant Λ_V is expressed through the conventional $\Lambda_{\overline{MS}}$ [18]:

$$\Lambda_V(n_f) = \Lambda_{\overline{MS}}(n_f)\exp\frac{a_1}{2\beta_0}, \quad a_1 = \frac{31}{3} - \frac{10}{9}n_f. \tag{5}$$

The important feature of the background coupling $\alpha_B(q)$ (and also $\alpha_B(r)$) is that it has correct perturbative limit at large q^2 (small r). Therefore in

BPT there are no additional (fitting) parameters and the $b\bar{b}$ spectrum and the wave functions are fully defined by the QCD constant $\Lambda_{\overline{MS}}(n_f)$ and the string tension (in BPT the pole mass of a heavy quark coincides with the conventional value [18]).

Due to the r-dependence of $\alpha_B(r)$ every $b\bar{b}$ state has its own characteristic coupling, denoted as $\alpha_{eff}(nL)$, which can be defined as

$$\alpha_{eff}(nL)\langle r^{-1}\rangle_{nL} = \langle \alpha_B(r) r^{-1}\rangle_{nL}. \tag{6}$$

Their values are smaller by $(20 \div 30)\%$ than the freezing value $\alpha_{cr} = \alpha_B(q^2 = 0)$ and grow for higher excitations (see Table 2).

Table 2: The effective coupling $\alpha_{eff}(nL)$ from [6]

state	1S	2S	3S	1P	2P	1D	2D	α_{cr}
$\alpha_{eff}(nL)$	0.41	0.45	0.46	0.50	0.51	0.54	0.54	0.60

The picture is different for the Cornell potential where $\alpha_V(r) = const \approx 0.4$ for all $b\bar{b}$ states [5].

It is convenient to consider the splittings between the spin-averaged masses $M_{cog}(nL)$ (instead of the absolute masses), in this way minimizing dependence of the splittings on the choice of parameters present in $V_{st}(r)$. It appears that the splittings $1D-1P, 1F-1P, 2D-2P$ do not practically depend on kinematics, see Table 3, where the solutions for relativistic Spinless Salpeter Equation (SSE) are compared to those for Schroedinger eq. From Table 3 one can also see that other splittings (like 2S-1P, 2P-1P) in NR case turn out to be by 6-10 MeV larger and this difference is much larger than the experimental error in a splitting, $\delta M \sim 1$ MeV.

Table 3: The splittings $\Delta = M_{cog}(n_1L_1) - M_{cog}(n_2L_2)$ (in MeV) in nonrelativistic (NR) case and for SSE (R case) [a])

splitting	NR	R	experiment
2S-1P	135	125	$\lesssim$ 123
2P-1P	377	370	360±1.2
3S-1P	111	103	$\lesssim$ 95
2D-1P	569	562	-
1D-1P	259	259	261.1± 2.2
2D-2P	192	192	-
1F-1P	462	462	-

[a]) The potential $V_B(r)$ in BPT is taken with $\sigma = 0.178$ GeV^2, $M_B = 0.95$ GeV, Λ_V(2-loop, $n_f = 5$) = 330 MeV, or $\Lambda_{\overline{MS}}$(2-loop, $n_f = 5$) = 242 MeV.

Our calculations show that the splittings $1D-1P, 1F-1P$, and $2D-2P$ do not practically depend on the variation of the quark pole mass, kinematics and weakly depend on the variation of the string tension. In [6] it has been shown that only the value of $\sigma = 0.177(3)$ GeV^2 provides good description of bottomonium spectrum as a whole. However these splittings appear to be very sensitive to the freezing value α_{cr} or to the QCD constant Λ.

This statement is illustrated by the numbers presented in Table 4 for three values of $\Lambda_V(n_f = 5) = 300$ MeV, 320 MeV, and 330 MeV which correspond to two-loop $\Lambda_{\overline{MS}}(n_f = 5) = 220$ MeV, 234 MeV, and 242 MeV, respectively.

Table 4: The splittings between spin-averaged masses in bottomonium (in MeV) for Spinless Salpeter Equation (R case) for the potential $V_B(r)$ with $\sigma = 0.178$ GeV^2, $m_b(pole) \cong 4.83$ GeV, $M_B = 0.95$ GeV.

splitting	$\Lambda_V^{(5)} = 300$ MeV, $\Lambda_{\overline{MS}}^{(5)} = 220$ MeV	$\Lambda_V^{(5)} = 320$ MeV, $\Lambda_{\overline{MS}}^{(5)} = 234$ MeV	$\Lambda_V^{(5)} = 330$ MeV, $\Lambda_{\overline{MS}}^{(5)} = 242$ MeV
1D-1P	252	257	259
1F-1P	450	457	461
2D-2P	188	190	192
2S-1P	123	124	125
3S-2P	103	103	103

From Table 4 it is clear that the splitting $\Delta = M_{cog}(1D) - M_{cog}(1P)$ turns out to be in good agreement with the experimental number, $\Delta(\exp) = 261.1 \pm 2.2(\exp)^{+1}_{-0}(th)$ MeV only for large value of the QCD constant $\Lambda_{\overline{MS}}^{(5)}$(2-loop) (experimental number for $M(1^3D_2) = 10161.1 \pm 2.2.$ MeV is taken from [19]). For $\Lambda_V^{(5)}$(2-loop)≈ 335 MeV the critical value of $\alpha_B(r)$ is large, $\alpha_{cr} = 0.60 \pm 0.01$ and corresponding $\Lambda_{\overline{MS}}^{(5)}$(2-loop)$\approx 240 \div 245$ MeV gives rise to $\alpha_s(M_Z) = 0.1193(2)$.

From this analysis we can predict $1F - 1P, 2D - 2P$ splittings (or the masses of $1F$ and $2D$ states) taking the same $\Lambda_V^{(5)}$ as for the $1D$ state. It gives

$$M_{cog}(1F) = 10362 \pm 2(\alpha_V) \pm 1(\sigma)\text{MeV} \tag{7}$$
$$M_{cog}(2D) = 10452 \pm 2(\alpha_V) \pm 2(\sigma)\text{MeV}$$

Since fine structure splittings of the $1^3F_J, 2^3D_J$ multiplets should be very small, as well as for 1^3D_J states [20], one can expect that the masses $M(1^3F_J)$ and $M(2^3D_J)$ have to be very close to the figures given in (7). Therefore the observation of the $1F, 2D$ states would be crucially important for the better understanding of the gluon-exchange term on the fundamental level.

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